:

- 1 -

•	1
A.	1
B.	5
C.	5
1.	5
2.	(error)
3.	6
4.	6
5.	(矯正)
D.	7
E.	7
•	8
A.	8
1.	8
2.	9
3.	가 가11
4.	
5.	16
B.	
1.	
2.	19
3.	25
C.	(Protocol methods)
1.	(Talking aloud method)27

2.	(Clinical Interview)2	27
3.	2	29
4.		!9
•	3	1
A.	3	1
B.	3	1
C.	3	3
1.		3
2.	3	3
D.	3	35
	3	6
•		Ü
	5	_
•	5	
A.	5	5
1.	(1)5	5
2.	(2)5	6
B.	7	9
1.	(1)	9
2.	(2)	0
		35
А.		
B.	8	8

		89
ABS	ΓRACT	92
<	A>	95
<	B>	100
<	C>	

7	- 1 >	<
19	- 1 >	<
21	- 2 >	<
23	- 3 >	<
24	- 4 >	<
25	- 5 >	<
55	- 1 >	<

PROTOCAL

37	(6)	1 > 10	1 >	<
37	3 (2)	2 >	2 >	<
38	3 (2)			
38	(6)	3 > 2	3 >	<
38		4 > 8	4 >	<
38	(1)	5 > 2	5 >	<
39	(2)	6 > 6	6 >	<
39	(3)	7 > 6	7 >	<
39	(4)	8 > 6	8 >	<
40	(5)	9 > 6	9 >	<
40	4 (1)	10 >	10 :	<
40	4 (1)			
41	(1)	11 > 1	11 :	<
41		12 > 12	12	<
42	(4)	13 > 11	13	<
42	(4)	14 > 3	14 :	<
43	(6)	15 > 10	15	<
43	(1)	16 > 11	16	<
43	(3)	17 > 2	17	<
43	(3)	18 > 2	18 :	<
44	(2)	19 > 3	19 :	<
44	(3)	20 > 11	20	<
44	(7)	21 > 10	21 :	<

45	10 (3)	1 >	<
45	10 (3)		
45	10 (3)		
45	(4)	2 >	<
46	(6)	3 >	<
46	(3)	4 >	<
46	(4)	5 >	<
46	(1)	6 >	<
47	(2)	7 >	<
47	(5)	8 >	<
47	(2)	9 >	<
47	(1)	10 >	<
47	(2)	11 >	<
48		12 >	<
48		13 >	<
48	(3)	14 >	<
48	(4)	15 >	<
49	6 (5)	16 >	<
49	6 (5)		
49	(1)	17 >	<
50	(3)	18 >	<
50	(2)	19 >	<
50		20 >	<
51	(1)	21 >	<
51	(2)	22 >	<

52	(4)	11	23 >	<
52	(3)	10	24 >	<
52	(2)	6	25 >	<
52	(5)	6	26 >	<
53	(4)	3	27 >	<
53	(5)	3	28 >	<
53	(1)	11	29 >	<
53	(5)	5	30 >	<
53	(4)	3	31 >	<

.

A. , . .

· 가

, 가 .

7 , , , 가 .

, 가 , , , ,

, ,

가 Erlwanger (1984) 가 Buyers 가 가 가 (1998). 가 가 가

가

(NCTM, 1989),

(Richard, 1986; Wagner,

- 12 -

1981).

가 ,

Clayton (1990)

가

가

가

가 , , , . .

가 가 .

, .

7 , .

.

,

7

B. 1. 1 2. 1 (C. 1. 가 가 (error) 2. г 가

- 15 -

가 .

:

·

: ,

.

:

:

, .

3.

1

4.

,

(x) (m)

< -1>

< -1>

x m	x m

5. (矯正)

,

D.

가

E.

1.

7 가 .

2.

Α. 1. (algebraic symbolism) 가 (algebraic 가 symbolism) . Harper (1987) 가 (Kieran et al, 1990). Diophantus (rhetorical stage, A.D. 250 (syncopated stage) Diophantus 16

- 18 -

(unknowns)

Vieta가

(symbolic algebra)

(givens)

(givens) 가 가 (procedural) (structural) ′ 가 가 (Wheeler, 1989). , 가 2. 230 + 320 = 550 - 150 =, 6 400 가

- 19 -

(G. Vergnaud, 1984). 12*m* 12 , 1 12 12*m* 가 12 가 . *m* . 1 (label) monkeys 4m가 가 . 가 가 " , 5 8 13 5 + 8 , a a + b

- 20 -

(procedural) .

가 가 a + b(process and object) (process-product dilemma) R. B. Davis . Collis (acceptance of lack of closure) 가 가 3. 가 가 가 가 가 (Diophantus) 1600 Vieta (indeterminate magnitude),

가 .

, , , 가 , 가

. 가 ' '

(2)

```
가
           3  n 11, y = x + 10
                                                          (simultaneous
representation)
                 가
          (identification number)
                                                      가
                          가
                                                                    , 3
     , 347
m n
(juxtaposition convention)
(Matz, 1979).
         3 m n 3, m, n
           가
 (3)
                           가
```

 $x^2 + 2x = 3$

가 \boldsymbol{x} (generality) (flexibility) 가 $S = \frac{1}{2} \cdot b \cdot h$ S(sum), b(base), h(height)(4) 가 3(x+2)+5=17-2xx가 가 가 가 , 가 가 가 가 (domain)

(freedom of delimitation property)'

(freedom of choice property)' 가 가 (generality) 가 . (flexibility) , 'X X 가 X 가 4. 가 가 가 가 5가 , 1988).

- 25 -

,

. . 가 .

, 가

5.

, *x*

. ,

-가

1 12 13 . Piaget

. , 가 가

- 가 가

,

- 27 -

1. Radatz(1979) 1925 Bu swell Judd가 30 Weimer (1925) $Seem\,ann\,(1925)$ 가 Gestalt theory 가 가 가 가 가 가 1970 B. Holtan & J. Dan Knifong(1976),

B.

Hendrik Radatz, Clements (1980) Newmann (1981),

N. M. Hadar & Zaslavsky(1987)

.

2.

B. Holtan & J. D. Knifong(1976) 6 35

MAT(Metropolitian Achievement Test)

< -1>.

< -1>

1.		
	12.5	3%
	61.5	13%
	88.5	19%
	80.5	17%
2.		
	22.5	5%
	30.5	6%
	58.5	12%
	85.5	18%
가	30.5	6%
	470	100%

Hendrik Radatz(1979)

가 (language difficulties) (difficulties in obtain information) (difficient mastery of prerequisite skills, facts, and concepts) (incorrect associations or rigidity of thinking). (application of irrelavant rules or strategies) Pippigg(1975) 가 1996. 가 Pippigg 가 Clements(1980) Newmann(1981) 5 7

- 2 >

< - 2 >

	Clements	Clements	NI array arr
	(1)	(2)	Newman
(Reading)	8	5	13
(Comprehension)	14	8	22
(Transformation)	27	25	12
(Process skill)	27	32	26
(Encoding)	2	2	2
(Careless)	22	28	25

7}
.
Casey(1980) (many step problems)

Newman Clements

. Casey

,

.

(Question form)

(Reading)
(Comprehension)

(Strategy selection)

(Skills selection)

(Skills manipulation)

N. M. Hadar & O. Zaslavsky(1987)

가 .

(1)	Misused Data):
A)	Misinterpreted language):
	(Logically invalid inference) : 가
	(Distorted theorem or definition): ,
,	(Unverified solution) : 가
(Techni	ical error) : ,
150 , 280	, 130 , , , , , , , , , , , , , , , , , , ,

- 32 -

< - 3 >	(%)				
	22	20			
	17	18			
	2	1			
	34	32			
	0	2			
	25	27			

(%)

< -4> (%)

	(%)
(Misused data)	16.8
(Misinterpreted language)	11.8
(Logically invalid inference)	4.6
(Misunderstood theorem or definition)	30.2
(Unmatched solution)	5.5
(Technical errors)	11.4
(Omission of solving process)	16.2
(Ambiguity of error)	3.5

< - 5 >

	1*	2*	3*	4*	5*	6*	7*	8*	
	3	2	13	9	5	2	8		42
	8	9		1	2	3	2	1	36
,	12	2	2	1	7	4	3		31
	2	12		1	3	11	5		36
	14	7	1	2		3	13	3	43
	2	2	1	39	1	3	2	1	51
	3			38	6	13	7		67
	2	2		39		1	6	3	53
		2		1	1	8	4	2	18
	13	2		1		1	5	4	26
	17		4	2		3	1	2	43
	1	12		4			4		21
									457

* : < - 4 >

3.

(1)

가 가

(2)

, (idea)

,

(1)

(2)

-가 가 ,

(3)

. 가

가 .

C.	(Protocol methods)		
protocol	methods(talking aloud procedure	.)	가
	(protocol methods)		
(written	protocol)		
(protoc	col)		
1.	(Talking aloud method)		
	,		
(protocol			•
· ·	,		
,			
		가	
	,		
	가 .		
2.	(Clinical interview)		

가

가 .

, ,

, , , ,

.

. " " . 5+5=10

가 5+5=?

, 가 .

(가). 가 3. 가 가 가 가 4. 가 가 가 가

- 39 -

(critical tests)

- 40 -

•

. A.

7 C 1 , 1 2 .

.

· 가 1 1 , 1 ' '

В.

1.

, ,

. 가 가 , 1 1

- 41 -

. , , 가

.

2.

45

1 , 2

, , 12

2 2

·

C.

(clinical)

1.

Casey 67 (, , ,) Polya 4 (, , ,)

1 40 , 42 297 ,

Nitsa Movshovitz Hadar Orit Zaslavsky가 6가 6가

2.

2

, Polya 가 가 2가 , 가 2 (written protocol) 가 가 (verbal protocol) 1:1

- 44 -

. 가

가

D.

.

2 가 2

•

•

, -

가 , , .

, . 4가

:

: , ,

· :

•

,

, 7

.

(1)

7 +
< 1 > 10 (6) (5 x + 3) - (4 x - 1)

= 8x - 3x = 5x.

< 2 > 3 (2) (a+b) \times (a+b) \times 4

= 4Qa+2b> = 8a+8b

$$<$$
 4 > 8
 yz
 $y = z$ $y \times z$ $y + z$ $25+26$ 25×26 $y-z$

$$<$$
 5 > 2 (1) $5x - 5$

$$(x^{2})$$
 (x^{2}) $(x^{$

$$\begin{array}{ccc}
< 7 > & 6 & (3) \\
& - x^2 \\
= + (+z)^2 = 4
\end{array}$$

< 8 > 6 (4)
$$x^{3} = -2^{3} = -8$$

< 9 > 6 (5)

$$|x+2|-|2x-3|$$

= $2(+2-1x-3)$
= $x-2x+2-3$
= $x-2x+2-3$
= $x-2x+2-3$

(COOX

$$<$$
 12 > 12
 A , B 7 A = 3 x + 2 , B = -7 x + 4 , 2 A + B

$$2(3x+2)+.(-1)x.+4$$

$$6x+4-1x+4$$

$$-x=-8$$

$$0=-1$$

$$b=-8$$

$$<$$
 13 > 11 (4)
 $\frac{10x - 15}{5} - \frac{12x - 8}{4}$

$$= 4(10 \times -15) - 5(12 \times -8)$$

$$= 40 \times -60 - 60 \times +40$$

$$= -20 \times -20$$

(2)

< 15 > 10 (6)

$$(5x + 3) - (4x - 1)$$

 $-$
 $-$
 $-$
 $-$
 $-$
 $-$
 $-$

< 16 > 11 (1)

$$2(x-1) + 3(x+1)$$

 $= 2x - 2 + 3x + 1$
 $= 5 \times -1$

$$<$$
 17 $>$ 2 (3)
 $a + 2(1 - a)$

$$= a + 2 - a = 2$$

(3)

< 18 > 2 (3)

$$a + 2(1 - a)$$

= $(2 - 2a)$

< 19 > 3 (2)

$$(a + b) \times (a + b) \times 4$$

$$<$$
 20 $>$ 11 (3) $4(x - \frac{1}{2}) - 3(\frac{x}{3} + 2)$

$$<$$
 21 > 10 (7) $0.4 x - 1.6 x$

(1)

$$10 (3)$$

$$(3 - a) + (3 + a)$$

$$= 3^{-} \alpha + 3 \alpha = 6 \alpha$$

10 (3)
$$(3-a) + (3+a)$$

$$= 3+3+0-0$$

$$6+0$$

< 2 > 10 (4)

$$x - 2y + 1 - \frac{2}{3}x + y$$

 $\times -\frac{2}{3}x - 2y + y + 1$
 $= -\frac{2}{3}x^2 - y + 1$

$$< 3 > 10 (6)$$

 $(5x + 3) - (4x - 1)$
 $= 3 \times - 3 \times = 5 \times$

< 4 > 2 (3)

$$a + 2(1 - a)$$

 $= 0^{2} + 2 - 1$

< 6 > 11 (1)

$$2(x-1) + 3(x+1)$$

 $-\frac{1}{2}x - \frac{1}{2} + \frac{1}{2}x + \frac{1}{3}$
 $-\frac{1}{6}x - \frac{1}{6}x - \frac{1}{6}x$

$$< 7 > 11 (2)$$
 $(-2 a - 1) - (-2 - 3 a)$

$$= -30 + -10$$

$$-40'$$

$$<$$
 9 > 3 (2)
 $(a+b) \times (a+b) \times 4$

$$<$$
 10 $>$ 2 (1) $5x - 5$

$$5x - x$$

5y

65+ y @ 5xy 652 y @ 5+5+5+5 @ y+y+y+y+y

< 13 > 9

30 FL (a+2)에 3을 곱한 것과 같은 것을 모두 고르시오. 첫 1,6 ① a+6 ②3x(a+6) ③ 3x2a ④ a+2 ⑤ 3a+6 ⑥ a+2×3

< 14 > 6 (3)
$$-x^{2}$$
= $t(+1) \times +(+2) = 4$

6 (5)
$$|-x+2|-|2x|^{2}-3|$$

$$|-x+2|-|2x|^{2}-3|$$

< 17 > 11 (1)
$$2(x-1) + 3(x+1) = 15x$$

$$2x = 17x$$

$$2x = 17x$$

$$2x = 17x$$

$$2x = 17x$$

$$<$$
 20 $>$ 1 (2) $?$ $?$ a , $?$ b

axb

< **21** > 12

$$2(3x+2)$$

$$(6x+4)+(-7x+4)$$

$$=(6x-7x)+(4+4)$$

$$=-x+8$$

(2)

< 22 > 11 (2)

$$(-2 a - 1) - (-2 - 3 a)$$

 $-2\alpha - 1 + 2 - 3\alpha (\overline{\alpha} - \overline{\beta})$
 $-2\alpha - 3\alpha - 1 + 2$

$$a \div 5 - (b - c) \div 4 = \frac{c}{7} - b + \frac{c}{4}$$

$$(3-a)+(3+a) = -a+a$$

(3)

$$<$$
 29 > 11 (1)
 $2(x-1) + 3(x+1)$
 $= 2x - 2 + 3x + 3$

(4)

$$< 31 > 3 (4)$$

 $a \div 5 - (b - c) \div 4$
 $= \frac{(b-5) 4}{25}$

.

A.

(,) . 40 , 42

. 가

,

. 40 297 , 42 866

< - 1>

< - 1 > (%)

58.2 %	25.3 %	16.5%	0%
60.8%	20.4%	13.6%	5.2%

,

,

. , A

1 1

< - 1>

, 58.2 % 60.8 %가 ' 가 가 , 25.3% 20.4%가

.

· 가

·

2. (2): 1() .

. 가

,

2 (-1 , -1)

(protocol)

C (Verbal Protocol)

< 1> .

(2)
$$\frac{1}{2}x + \frac{2}{3}x$$

$$(3) (5x + 3) - (4x - 1)$$

```
(4) (2 a - 3 b) + (4 a + 5 b)
(5) a + a
(6) a x a
                       가?
:
                        가
:
: (
                 가?
                                       가?
                                                               ...
                                        가?
         가
                                   •••
: (
                       가 ?[_
  a - 3 a = -3
                       ?
                                                            ?
                                              -3
:
                          )
            a
                                               -3
                          a
                              ?
         a가
                                                                 ?
: a
```

: • • •

```
?
                     ?
1
                        가
     가
: a 1 .
: 1 - 3·1 = 1 - 3 = - 2 가 .
: , a 2
: 2 - 3 \cdot 2 = 2 - 6 = -4.
                가 a - 3 a
: .
      a + a
: 2 a
: 2 a가 ?
···· ( ) .
: 1 + 1 a + a
: 1 a
: , a
1 a
```

```
: 7t a - 3 a .
a a가 , - 3 a - a가
: , - 3 a - a7 t 3 .
: . .
가
          ?
:
    가
    2 a + 3 a^2 + 3 a + 2 a^2
                  2 a \quad 3 a , 3 a^2
2 a^2
                2 3 5 5 a가
              5 a^2
   , 3 2
: .
                   -2 a가 .
a - 3 a 1 3
: , . . .
             ?
: .
        가
: (
      가)
```

```
: 가
 : 가
                                                 가?
 \frac{1}{2}x \pm \frac{2}{3}x \equiv \frac{1}{6}x \pm \frac{2}{6}x = (\frac{1}{6} + \frac{2}{6})x = \frac{3}{6}x \quad [\underline{\qquad}]
\frac{1}{2}x \pm \frac{2}{3}x \equiv \frac{3}{6}x \pm \frac{4}{6}x \equiv (\frac{3}{6} \pm \frac{4}{6})x \equiv \frac{7}{6}x
 · , !
 : (
```

```
?
         가
: .
          3
: (
                  ?
: (5x + 3) - (4x - 1) = (5x + 3) - 4x - 1 [____]
                = 5x - 4x + 3 - 1
                = x + 2
: ,
                    ?
                 가
                                     ?
: (
             가)
: , 1
: 가
: • • •
                                     ?..
                         ?
: !
 (5x + 3) - (4x - 1) = (5x + 3) - 4x + 1
                = 5x - 4x + 3 + 1
                = x + 4 [ ]
```

```
: 가
                           ?
: . 가
     4
                   ?
: (
                                   ?
                               ?
: (
               )
(2 a - 3 b) + (4 a + 5 b) = (2 a - 3 b) + 4 a + 5 b
                       = (2 a + 4 a) + (-3 b + 5 b)
                       = 6 a + 2 b
                   가
                                         가
                       가
                                                ?
                                가 ?
              가)
                        가
: (
            ?
:
          (2x + 1) - (3x + 2)
: (2x + 1) - (3x + 2) = (2x + 1) - 3x - 2
                    = (2x - 3x) + (1 - 2)
                    = x - 1
```

?

:

```
..) <u>- x - 1</u> ...
: ?, (
: !
 !
5
: \underline{a + a \equiv a^2} \left[ \underline{\phantom{a}} \right]
: . . .
: a a
              !
: ! 2 a . [ ]
: ?
가
: . .
 6
: <u>a × a = 2 a</u> (_____)
1 1
 \underline{a} \times \underline{a} = \underline{a}^2 . [
: .
5 6
2 a × a
: 2 a \times a = 2 \times a \times a = 2 a^2 !
: , ! , 2 a + a ?
: 3 a .
: !
가
                      ?
: .
```

```
:
< 2> x = -3 ,
(1) x^2
(2) - x<sup>2</sup>
(3) x^3
(4) \mid x + 3 \mid - \mid 2x - 3 \mid
 :
 : (
                                 ? 1
  : , x
  : x = -3 , x^2 = x \times x = (-3) \times (-3) = 9 .
  : !
            가
   1
  : (
                                     가
                     x = -3
  : ! .
```

?

```
: 2
 : \underline{-x^2} = \underline{-(-3)x-(-3)} = \underline{(+3)x(+3)} = \underline{9} [\underline{}]
 : 	 ? - x^2   x^2
                                   가
 -x^2 = -(-3) \times (-3) = -(+9) = 9 [_____]
 : 9가 ?
 : !
-x^2 = -(-3) \times (-3) = -(+9) = -9
: • • •
:
: , .
-x^2 = -(-3)^2 = -9 7 ?
       ? 가
     3
: .
```

```
x^3 = (-3) \times (-3) \times (-3) = (+9) \times (-3) = -27
: . !
                      가
                                   ?
: x^3 = (-3)^3 = -27
             ?
                     가
                                 가 ?
|x + 3| - |2x - 3| = |-3 + 3| - |2 \cdot (-3) - 3|
                   = 0 - | -6 - 3 | = 9  [ _____]
: !
                    가
   ?
: !
|x + 3| - |2x - 3| = |-3 + 3| - |2 \cdot (-3) - 3|
                   = 0 - | -6 - 3 | = -9 [
: .
```

```
: 가
      3>
(1) (3 - a) + (3 + a)
(2) 2 \cdot (x - 1) + 3 \cdot (2x + 5)
(3) (-3a-2) - (-3-2a)
(4) 4(x - \frac{1}{4}) - 3(\frac{x}{3} + 1)
 : (
                                   ?
  : , 1
  : (3 - a) + (3 + a) = -3 a + 3 a = 0 [ ____ ]
  : 가
  : 3 - a
```

(3 - a) + (3 + a) = (3 - a) + 3 + a

= 6 + (-a + a) [_____]

= (3 + 3) + (-a + a)

```
?
-a + a
                        ?
: (
               ) , 0
                       가
-1 + 1
               ? 0
                0
              6____
                         ]
            !
                                     !
                              ?
: 2
: 2(x - 1) + 3(2x + 5) = (2x - 2) + (6x + 15)
                    = (2x + 6x) + (-2 + 15)
                    = 8x + 13x
                                                   ?
: 1
          3
: (
```

```
: (-3 \ a \ - \ 2) \ - \ (-3 \ - \ 2 \ a) \ = \ (-3 \ a \ - \ 2) \ + \ 3 \ - \ 2 \ a [_____]
                      = (-3a - 2a) + 3 - 2
                      = -5 a + 1
             ?
                    가
                                                ?
  !
                                                             ?
: (
: 가
                                              가
 (-3a-2)-(-3-2a)=(-3a-2)+3+2a
                      = (-3 a + 2 a) + 3 - 2
                      = -a + 1
       !
                                                     !
```

```
: (
: ×
: (x - y) \times (-1) = (-1) \times (x - y)
                       = (-1)(x - y) = -x - y [
: (
: 1
                                 가
                                                        가
                                                 -1
                                       \boldsymbol{x}
 \boldsymbol{x}
: , y
                      ?
: , . 가
: (x - y) \times (-1) = (-1) \times (x - y)
                 = (-1)(x - y)
                 = \underline{-x} \underline{+y} [
                         가
                                                               ?
```

```
가
: \underline{a \div 5} - (b - \underline{c}) \div 4 = \underline{a} - b + c \div 4
                          = \frac{a}{5} - b + \frac{c}{4}
                          = \frac{a}{5} - b + \frac{c}{4} \times 20
                           = 4 a - 20 b + 5 c  [_____]
: +, - ×, ÷가
                                         ?
: × ÷
: (
```

```
: a \div 5 - (b-c) \div 4 = \frac{a}{5} - (b-c) \div 4
                                      = \underline{\frac{a}{5}} = \underline{\frac{(b-c)}{4}}
                                      = \underline{\frac{a}{5}} = \underline{\frac{(b-c)}{4}} \times \underline{20} \left[ \underline{\phantom{a}} \right]
                                       = 4 a - 5(b - c)
                      가
                                                                                                               ?
        20
                        가
     가
                                                 가
                                                                                                                 가
                    가
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                                    가
```

, 가 가 .

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: $a \div 5 - (b - c) \div 4 = \frac{a}{5} - (b - c) \div 4$ $= \frac{a}{5} - \frac{(b - c)}{4} \quad \boxed{1}$

?

?

: 3: ?

: . . .

 $: \underline{x \div 3 \times x} = \underline{x \times \frac{1}{3} \times x} = \underline{\frac{2}{3}} \times \underline{\qquad \qquad }$ $: \underline{\qquad \qquad }$

$$\frac{x \div 3 \times x}{2} \times \frac{1}{3} \times x = \frac{x}{3} \times$$

$$: \frac{x}{3} \qquad x$$

$$x \div 3 \times x \equiv x \times \frac{1}{3} \times x \equiv \frac{x}{3} \times x \equiv \frac{x^2}{3}$$

:
$$a \times (-\frac{1}{2}) \div b = a \times (-\frac{1}{2}) \times \frac{1}{b}$$

= $(-\frac{1}{2}a) \times \frac{1}{b}$

$$= \frac{-\frac{1}{2}a}{b} \left(\frac{1}{2} \right)$$

$$a \times (-\frac{1}{2}) \div b = a \times (-\frac{1}{2}) \times \frac{1}{b}$$

$$= (-\frac{1}{2}a) \times \frac{1}{b}$$

$$= -\frac{a}{2} \times \frac{1}{b} = \underline{-\frac{a}{2b}} \quad \boxed{ }$$

: 가

 $\begin{array}{c} \vdots \quad a \ \div b \qquad ? \\ \vdots \quad \frac{a}{b} \quad . \\ \vdots \quad 7^{\dagger} \qquad \vdots \\ \vdots \quad ! \qquad . \\ \underbrace{(x = 2) \div 3 \times y}_{3} = \frac{(x - 2)}{3} \times y \equiv \frac{(x - 2)}{3} y \quad \boxed{1} \\ \vdots \quad . \qquad ? \\ \vdots \quad . \qquad . \\ \vdots \qquad . \qquad ? \\ \end{array}$

$$<$$
 5> $a = 2$, $b = -3$.

(1)
$$2(a - b)$$

(2)
$$3 a + 4 b$$

(3)
$$a^2 - 3 a b$$

(4)
$$-a^2 + b^2$$

(5)
$$| b - 2a | - | 1 - b^2 |$$

:

: (

: ? 가?

: a b .

: . . .

$$: \underline{2(a-b)} = \underline{2} \underline{a} - \underline{2} \underline{b} = \underline{4} - \underline{6} = -\underline{2}$$
 [________

```
가
:
: (
                        ) .
       가
:
 2(a - b) = 2 a - 2 b
             = 2 \cdot 2 - 2 \cdot (-3)
             = 4 + 6 = 10
                                 .[ ]
                                                    ?
                               가
                       가
                                                                                              가
                                                  ?
                2
                                            ?
: 2
: 1
                                                                          ?
: \underline{3} \ a + \underline{4} \ b = \underline{6} + \underline{12} = \underline{18} \ [\underline{\phantom{0}}]
                               가
                                   가 ?
                   가
```

```
: !
   \underline{3} \ a \ + \ \underline{4} \ b \ = \ \underline{3 \cdot 2} \ + \ \underline{4 \cdot (-3)} \ = \ \underline{6} \ + \ (-12) \ = \ \underline{6} \ - \ \underline{12} \ = \ - \ \underline{6}
 : 가
                                                                ?
                                                               ?
 : \quad a^2 - 3 \ a \ b = 2^2 - 3 \cdot 2 \cdot (-3)
                            = 4 - 6 \cdot (-3)
                             = 4 + 18 = 22 [ ]
 : (
                            ) .
                                      ?
```

: $a^2 + b^2 = (-2) \times (-2) + (-3) \times (-3)$

```
= (+ 4) + (+ 9) = + 13 = 13  [_____]
:
           ) .
: (
         . 가 - a<sup>2</sup>
: !
                                          ?
: • • •
: 가 - a<sup>2</sup>
  a^2
: a^2 \qquad a \times a \qquad .
: a 2
: 2 \times 2 = 4.
: . a 가 - 2
: a^2 = a \times a = (-2) \times (-2) = +4 = 4
: . .
a가 a^2 가 ?
: (
?
                                        ?
: !
: , a^2 = +4 .
: . . - a<sup>2</sup>
: ( ) a<sup>2</sup>
                                   가 ?
: .
-a^2 + b^2 = -4 + (+9) = -4 + 9 = 5 . [
```

: . .

```
: -a^2
         가
: 5
                                        ?
: .
| b - 2 a | - | 1 - b^2 | = | -3 - 2 \cdot 2 | - | 1 - (-3)^2 |
                    = | - 3 - 4 | - | 1 - (+9) |
                    = | -7 | - | 1 - 9 |
                    = 7 + 8 = 15 (_____)
                                  가
          | - 2 |
                                ?
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: 2 .

B. 1. (1) 1 1 '가 가 가 가 1 가

- 86 -

가 가 . . + ×

, + **x** . 2 a+3 5 a , 2 a+3 b 5 a b .

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- 88 -

 x^2 x^3
 x^2 x^3

가 가 .

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(1) ,

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(2) , 가

(3)

(4) 12 , 45 .

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1.

 $(1) \quad n \qquad \qquad x \qquad \qquad 1$

(2) 가 가 a, 가 b

(3) 130 a 1000

2. .

 $(1) \ 5 x - 5$

(2) 5x - x

(3) a + 2(1 - a)

(4) a + a

(5) $a \times a$

(6) x - 3x

 $(7) \ 3 \ n + (5 \ m + 4 \ n)$

3. \mathbf{x}, \div

 $(1) -1 \times x \times y$

(2) $(a + b) \times (a + b) \times 4$

(3) $a \div 2 \times b \div c$

(4) $a \div 5 - (b - c) \div 4$

(5)
$$a \times (-\frac{1}{2}) \div b$$

5.
$$x = 3$$
, $y = -4$

(3)
$$3x - 4y$$

(4)
$$2(x - y)$$

(5)
$$5x - \frac{1}{3}y$$

6.
$$x = -2$$

$$(1) - x + 5$$

(2)
$$x^2$$

(3) -
$$x^2$$

- (4) x^3
- (5) | x + 2 | | 2x 3 |
- 7. 5*y* . .
 - 5+y 5xy 5 y 5+5+5+5 y+y+y+y
- 8. *yz*
 - y z $y \times z$ y+z 25+26 25×26 y-z
- 9. *a* +2 3
- a+6 3x(a+6) 3x2a a+2 3a+6 $a+2\times 3$
- 10.
 - (1) 2 a + 5 a
 - (2) -2 a + 4 a
 - (3) (3 a) + (3 + a)
 - (4) $x 2y + 1 \frac{2}{3}x + y$
 - (5) 2a + 5b + a
 - (6) (5x + 3) (4x 1)
 - (7) 0.4 x 1.6 x

11. .

$$(1) \ 2(x - 1) + 3(x + 1)$$

(3)
$$4(x - \frac{1}{2}) - 3(\frac{x}{3} + 2)$$

$$(4) \quad \frac{10x - 15}{5} \quad - \frac{12x - 8}{4}$$

12.
$$A, B \nearrow A = 3x + 2, B = -7x + 4,$$

 $2A + B$ $ax + b$. $a + b$

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< B>

(1)

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(4)

(5)

(6) 가 ,

1. .

(1)
$$-2 a + 4 a$$

(2)
$$\frac{1}{2}x + \frac{2}{3}x$$

$$(3) (5x + 3) - (4x - 1)$$

$$(4) (2 a - 3 b) + (4 a + 5 b)$$

$$(5) a + a$$

2. x = -3 .

(1)
$$x^2$$

(2)
$$-x^2$$

(3)
$$x^3$$

$$(4) \mid x + 3 \mid - \mid 2x - 3 \mid$$

3.

$$(1) (3 - a) + (3 + a)$$

$$(2) \ 2(x - 1) + 3 \ (2x + 5)$$

$$(3) (-3 a - 2) - (-3 - 2 a)$$

(4)
$$4(x - \frac{1}{4}) - 3(\frac{x}{3} + 1)$$

$$(5) \quad \frac{15x - 10}{5} \quad - \quad \frac{8x - 12}{4}$$

4. ×, ÷

(1)
$$(x - y) \times (-1)$$

(2)
$$(x - 2y) \times 3$$

(3)
$$a \div 5 - (b - c) \div 4$$

(4)
$$(x - 2) \div 3 \times y$$

(5)
$$a \times a \times (-3) \times b$$

5.
$$a = 2$$
, $b = -3$

$$(1) \ 3 \ a \ + \ 2 \ b$$

(2)
$$2(a - b)$$

(3)
$$a^2 - 3 a b$$

(4)
$$-a^2 + b^2$$

(5)
$$| b - 2a | - | 1 - b^2 |$$

< C>

1.		•			
2.		가	?		
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13.				가	가?
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